MAGNETIZED PERFECT FLUID BIANCHI TYPE-III COSMOLOGICAL MODEL WITH VARIABLE $\Lambda$ AND $G$

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ABSTRACT
In the present study, we have studied Bianchi type-III cosmological model with perfect fluid source containing magnetic field in general theory of relativity with linearly varying variable cosmological and Gravitational constant. We have obtained the general solutions of the Einstein's field equations for the cosmological model by assuming the circumstance of anti-stiff fluid i.e. relation between pressure and density and observed that in our derived model the Universe represents shearing, non-rotating and expanding model of the universe with big-bang starts in the midst of both scale factors is monotonically increasing function of time. The behavior of the Universe in presence of magnetic field and singularities in the model are discussed in detail. Furthermore some physical and geometrical aspects of the model are discussed.

KEYWORDS
Bianchi type-III Cosmological model; anti-stiff fluid; variable cosmological and Gravitational constant.

1. INTRODUCTION
It is interesting to note that magnetic field present in galactic and intergalactic spaces play a significant role at cosmological scale. The study of magnetic field in the matter distribution is of considerable interest as it provides an effective way to understand the initial phases of cosmic evolution. The inclusion of the magnetic field is motivated by the observational cosmology and astrophysics indicating that many subsystems of the universe possess magnetic fields. A cosmological model containing a global magnetic field is necessarily anisotropic. An understanding of the effect of a magnetic field upon the dynamics of the universe is necessary during early and late time evolution of the universe. During the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled to the gravitational field and subsequently forming neutral matter during expansion of the universe [1]. Bali and Meena [2] investigated magnetized stiff fluid tilted universe for perfect fluid distribution in general relativity. Banerjee and Banerjee [3] studied stationary distribution of dust and electromagnetic fields in general relativity. Banerjee et al. [4] have investigated an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. Recently, Bali and Upadhyay [5] have investigated LRS Bianchi Type I strings dust-magnetized cosmological models. [6-12] are some of the authors who have studied the cosmological models with magnetic field and have pointed out its important in the early evolution of the universe.

An awesome abundance of observational evidence (SNe-Ia Supernova, CMBR, LSS and WMAP) favor the universe is spatially flat and late-time accelerating expansion which is not fit within the framework of Einstein's General Theory of Relativity. The proposals that have been put forward to explain this observed phenomenon can basically be classified into two categories. One, an exotic component with negative pressure called mysterious energy or Dark Energy (DE) introduce into Einstein's General Theory of Relativity (GTR). The dynamical dark energy models are classified into two different categories: (a) the scalar fields including Quintessence, Phantom, Quintom, K-essence, Tachyon, Dilaton, and so forth. (b) The interacting models of dark energy such as the Chaplygin gas models, Braneworld models, Holographic and Agegraphic models. For a good review of the dynamics of different dark energy models, Mishra and Biswal [13] have constructed a self-consistent system of Bianchi Type-VI, cosmology in five dimensions with a binary mixture of perfect fluid and dark energy where the dark energy is chosen to be either the quintessence or Chaplygin gas using solutions to the corresponding Einstein's field equations as a quadrature and observed that the equation of state parameter for dark energy is found to be consistent with the recent observations (SNe-Ia Supernova with CMBR) and Galaxy Clustering Statistics. Chirde and Shekh [14, 15] investigated interacting two-fluid viscous dark energy and magnetized dark energy cosmological models in self-creation cosmology and Lyra geometry respectively. The same authors [16, 17] studied plane-symmetric dark energy cosmological model in the form of wet dark fluid in modified gravity. Also same observations indicate that our universe is flat and currently consists of approximately 2/3 DE and 1/3 dark matter source. In recent years, many authors [18-30] have shown keen interests in studying the DE universe with various contexts. However, the simplest and most theoretically appealing candidate for DE is vacuum energy (or the cosmological constant $\Lambda$) with a constant equation of state parameter equal to 1−. Among many possible alternatives, the simplest and most theoretically appealing possibility for dark energy is the energy density stored on the vacuum state of all existing fields in the universe, i.e., $\rho = \frac{\Lambda}{8\pi G}$. However, a constant $\Lambda$ cannot explain the huge difference between the cosmological constant inferred from observation and the vacuum energy density resulting from quantum field theories. Recent observations indicate that $\Lambda \sim 10^{-55} \text{cm}^{-3}$ while the particle physics prediction for $\Lambda$ is greater than this value by a factor of order $10^{9}$. This discrepancy is known as the cosmological constant problem. The simplest way out of this problem is to consider a varying cosmological term, which decays from a huge value at initial times to the small value observed in these days in an expanding universe [31, 32].

2. Metric, Energy momentum tensor and Field equations:
FRW models are homogeneous and isotropic. These isotropic models are unstable near the origin and fail to describe the early universe, but anisotropy plays a significant role in the models near $t = 0$, hence spatially homogeneous and anisotropic Bianchi type models are undertaken to study the universe at its early stage of evolution. Moreover, these models help in obtaining more general cosmological models than the isotropic FRW models. Out of the different Bianchi type models, the LRS Bianchi-III model is more interesting to explain the phenomenon of the universe. Hence, we consider a spatially homogeneous Bianchi type-III cosmological model of the form

$$ds^2 = dt^2 - R^2(t)dr^2 + S^2(t)(d\theta^2 + d\phi^2)$$  \hspace{1cm} (1)

where the metric potentials $R$ and $S$ are the functions of time $t$.

To discuss the kinematics of the universe, we need to define some kinematical parameters of the universe which has a great importance in cosmology.

Spatial volume and the scale factor are

$$V = a^3 = (RS)^3$$  \hspace{1cm} (2)

The mean Hubble's parameter, which expresses the volumetric expansion rate of the universe, given as

$$H = \frac{1}{3V} \left[ -\frac{1}{3} \left( \frac{\dot{R}}{R} + 3 \frac{\dot{S}}{S} \right) \right]$$  \hspace{1cm} (3)

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Another important dimensionless kinematical quantity is the mean deceleration parameter, which tells whether the universe exhibits accelerating volumetric expansion or not is

$$q = -1 + \left( \frac{1}{H^2} \right)$$

for \(-1 < q < 0\) and \(q = 0\) the universe exhibit accelerating volumetric expansion, decelerating volumetric expansion and expansion with constant-rate respectively.

To discuss whether the universe either approach isotropy or not, we define an anisotropy parameter of the expansion as

$$A_a = \left( \frac{\Delta H}{H} \right)^2$$

where \(\Delta H = H_0 - H\)

The scalar expansion and shear scalar are defined as

$$\theta = \frac{R_4}{R} + \frac{S_4}{S}$$

and

$$\sigma^2 = \frac{3}{2} A_m H^2.$$  

The energy momentum tensor for perfect fluid in presence of magnetic field is given by

$$T^j_i = (p + \rho) v^j v^i - pg^i_j + E^j_i,$$

where \(\rho\) is the energy density, \(p\) be the isotropic pressure, and \(E^j_i\) is the electromagnetic field given as [30]

$$E^j_i = \rho \left[ \left( v^j v^i + \frac{1}{2} g^j_i \right) - h_i h^j \right],$$

with

$$h_i = \sqrt{\frac{-g}{2\mu}} \epsilon_{ijkl} F^{kl} v^j \left| g^i \right|^2 = h_i h^i.$$  

where \(h_i\) be the magnetic flux vector, \(F^{ij}\) be the electromagnetic field tensor, \(\epsilon_{ijkl}\) be the Levi-Civita symbol, \(\mu\) be the magnetic permeability, and \(v^i\) be the flow velocity satisfying

$$g_{ij} v^j v^i = 1.$$  

We assume that the coordinates to be comoving so that

$$v^1 = 0, v^2 = v^3, v^4 = 1.$$  

The incident magnetic field is taken along x-axis so that

$$h_1 = 0, h_2 = 0, h_3 = 0, h_4 = 0.$$  

Assume that there is a magnetic field along x-direction. Hence, \(F_{23}\) is the only non-vanishing component of \(F_{ij}\).

Maxwell's equation gives

$$F_{ij,k} + F_{jk,l} + F_{lk,i} = 0$$

which leads to

$$F_{ij} = \text{constant} = \mu (\text{say}).$$

(15)

where \(\mu\) is a constant characterizing the magnetic field intensity and \(F_{14} = 0 = F_{24} = F_{34}\) due to the assumption of infinite electrical conductivity (Roy Maartens [31]).

It follows from equation (10) that the non-zero component of magnetic flux vector is

$$h_i = \frac{R \mu}{\mu S^2} \sinh \theta.$$  

Thus using (16) into (9), the non-trivial components of \(E^j_i\) are given by

$$E^1_i = -E^2_i = \frac{H^2}{2} \sinh \theta.$$  

Using above condition presented in equations (27), the equations (20) and (21), takes the form

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} = \frac{R_{44}}{R} + \frac{S_{44}}{S}.$$  

### 3. Solution of the Field equations:

Equations (19) to (21) are the system of three linearly independent equations in six unknowns \(\Lambda, \alpha, \beta, \Gamma, \Omega^2\), and \(G\). Therefore, in order to fully determine the system, we use the constraining equation as:

1. The comment of velocity red-shift relation for extragalactic sources recommends that Hubble's expansion of the universe is isotropy within \(\approx 30\) percent today. To put more precisely, red-shift studies place the limit \(\alpha / H < 0.3\) on the ratio of shear \(\alpha\) to Hubble constant \(H\) in the neighborhood of our galaxy today. Latter on Collin et al. [33] have pointed out that for spatially homogeneous metric; the normal congruence to the homogeneous expansion satisfies that the condition \(\alpha / (0r)\) is constant. It gives

$$S = R^n,$$

where \(n\) is any constant.

2. The fluid in the model is anti-stiff fluid i.e.

$$p + \rho = 0.$$  

(27)

3. Consequences of the following case of the phenomenological decay of \(\Lambda\) i.e.

$$\Lambda \approx H^2.$$  

(28)

The different dynamical laws proposed for the decay of \(\Lambda\) have been widely studied by Chen and Wu [34], Carvalho et al. [35], Schutzhold [36], Vishwakarma [37] to name only a few.

Using above condition presented in equations (27), the equations (20) and (21), takes the form

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} = \frac{R_{44}}{R} + \frac{S_{44}}{S} = M S.$$  

(29)
where $M = -\frac{k^2}{4\pi \sinh^2 \theta}$.

Using equation (23), equation (29) becomes

$$R_{44} = \frac{2n}{n+1} R + \frac{R^{2n+1}}{(n+1)^2} = R^{4n}.$$  \hfill (30)

Consider, $R_q = f(R)$ and $R_{44} = f(R)^q(R)$. 

$$\frac{d}{dR} f'(R)^2 - \frac{4n}{(n+1)R} f(R)^2 + \frac{2}{(n+1)R^{2n+1}} = R^{4n}.$$  \hfill (31)

Above equation gives

Using above equation (32) together with the relation (23), the spatially homogeneous and anisotropic Bianchi type-III cosmological model takes the form

$$f^2 = R^2 = -\frac{M(n+1)}{(-2n^2 - 3n + 1)}$$

$$+ \frac{n(n+1)}{(2n^2 + 3n - 1)} R^{4n} - \frac{1}{(n^2 - 2n + 1)} c R^{4n}.$$  \hfill (32)

The Kinematical parameters which are important for describing the geometrical behavior of the universe are,

5. Kinematical Parameters:

The spatial volume and average scale factor,

$$V = \frac{M(n+1)}{(-2n^2 - 3n + 1)} R^{4n} + \frac{1}{(n^2 - 2n + 1)} c R^{4n}.$$  \hfill (33)

The Hubble Parameter,

$$H = \frac{M(n+1)}{(-2n^2 - 3n + 1)} R^{4n} - \frac{1}{(n^2 - 2n + 1)} c R^{4n}.$$  \hfill (34)

The Expansion scalar,

$$\theta = 3 \left[ \frac{M(n+1)}{(-2n^2 - 3n + 1)} R^{4n} - \frac{1}{(n^2 - 2n + 1)} c R^{4n} \right]^{\frac{1}{2}}.$$  \hfill (35)

6. CONCLUSION:

In the present study, we have studied Bianchi type-III cosmological model with perfect fluid source containing magnetic field in general theory of relativity with linearly varying variable cosmological and Gravitational constant by obtaining the exact solution of field Einstein's field equations for the cosmological model assuming the circumstance of anti-stiff fluid and observed that in our derived model the universe represents shearing, non-rotating and expanding model of the universe with big-bang starts in the midst of both scale factors is monotonically increasing function of time.

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